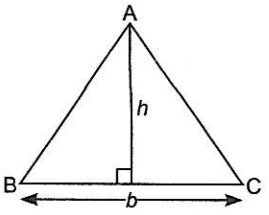
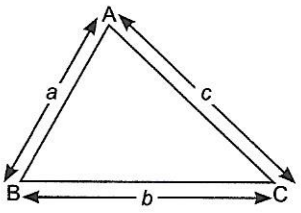
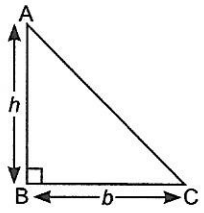
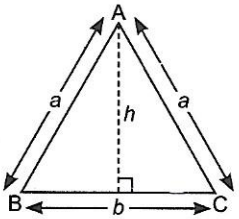
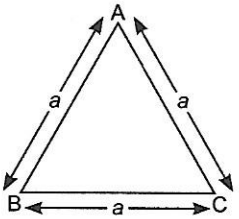
HERON'S FORMULA

12

Basic Concepts Related to Area of a Triangle

- **Area:** The measurement of extent of surface is called its area. It is measured in square metre (m^2) or square centimetre (cm^2).
- **Area Related to Triangle:**

(*i*) Triangle (Altitude and base is given)		Area $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times b \times h$
(*ii*) Triangle having sides of measure a, b and c.		• Perimeter $= a + b + c$ • Semi-perimeter(s) $= \frac{a+b+c}{2}$ • Area $= \sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)
(*iii*) Right-angled triangle.		Area $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times b \times h$
(*iv*) Isosceles triangle (Base $= b$, Equal sides $= a$)		Height of triangle, $h = \sqrt{a^2 - \frac{b^2}{4}}$ Area $= \frac{1}{2} \times b \times h$ $= \frac{1}{2} \times b \times \sqrt{a^2 - \frac{b^2}{4}}$ $= \frac{1}{4} b\sqrt{4a^2 - b^2}$
(*v*) Equilateral triangle (All sides of equal length)		Area $= \frac{\sqrt{3}}{4} \times (\text{Side})^2 = \frac{\sqrt{3}}{4} \times a^2$

SOLVED QUESTIONS BASED ON EXERCISE 12.1

Very Short Answer Type Questions [1 Mark]

1. Find the area of triangle having base 6 cm and altitude 8 cm. [CBSE 2011]

Sol. Area of triangle $= \frac{1}{2} \times$ base $\times$ altitude

$$= \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

2. Find the area of triangle whose sides are 13 cm, 14 cm and 15 cm.

Sol. Given $a = 13$ cm, $b = 14$ cm and $c = 15$ cm

Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{13+14+15}{2} = \frac{42}{2} = 21$ cm

Using Heron's formula,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{21(21-13)(21-14)(21-15)}$$

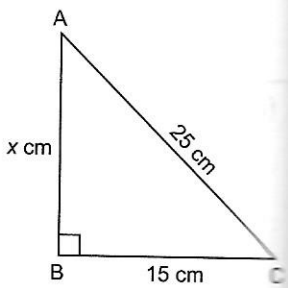
$$= \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7 \times 3 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3}$$

$$= \sqrt{\underline{7 \times 7} \times \underline{3 \times 3} \times \underline{2 \times 2} \times \underline{2 \times 2}}$$

$$= 7 \times 3 \times 2 \times 2 = 84 \text{ cm}^2$$

3. If the base of a right-angled triangle is 15 cm and its hypotenuse is 25 cm, then find its area.

Sol. Using Pythagoras theorem in right-angled ΔABC, we have



$$AC^2 = AB^2 + BC^2$$

$\Rightarrow$ $25^2 = x^2 + 15^2$

$\Rightarrow$ $x^2 = 25^2 - 15^2 = 625 - 225 = 400$

$\therefore$ $x = \sqrt{400} = 20$ cm

$\therefore$ Area of right-angled triangle $= \frac{1}{2} \times$ base $\times$ height

$$\text{ar}(ABC) = \frac{1}{2} \times 15 \times 20 = 150 \text{ cm}^2$$

4. Two sides of a triangle are 13 cm and 14 cm and its semi-perimeter is 18 cm. Find the third side of this triangle. [CBSE 2011]

Sol. Semi-perimeter of triangle, $s = \frac{a+b+c}{2}$

$\Rightarrow$ $18 = \frac{13+14+c}{2}$

$\Rightarrow$ $c = 36 - 27 = 9$

$\therefore$ Third side of the triangle is 9 cm.

5. Find the area of an equilateral triangle with side $2\sqrt{3}$ cm.

Sol. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

$$= \frac{\sqrt{3}}{4} \times (2\sqrt{3})^2 \qquad \text{(Given side} = 2\sqrt{3} \text{ cm)}$$

$$= \frac{\sqrt{3}}{4} \times 4 \times 3 = 3\sqrt{3} \text{ cm}^2$$

Short Answer Type Questions I [2 Marks]

6. Find the area of an isosceles triangle whose one side is 10 cm greater than each of its equal sides and perimeter is 100 cm. **[CBSE 2014]**

Sol. Let equal sides of an isosceles triangle be x cm.

Therefore, the length of its greater side = $(x + 10)$ cm

Perimeter of an isosceles triangle = 100 cm

$\Rightarrow$ $x + x + (x + 10) = 100$

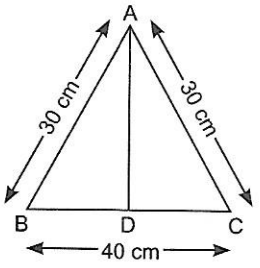
$\Rightarrow$ $3x = 100 - 10 = 90$

$\Rightarrow$ $x = \frac{90}{3} = 30$ cm

$\therefore$ Base of an isosceles triangle = $10 + x = 10 + 30 = 40$ cm

We know that in an isosceles triangle, its altitude bisects the base.

$\therefore$ ΔABD is a right-angled triangle.



$\Rightarrow$ $AB^2 = BD^2 + AD^2$ (Using Pythagoras Theorem)

$\Rightarrow$ $30^2 = 20^2 + AD^2$ ($\because$ BD $= \frac{1}{2}$BC $= \frac{1}{2} \times 40 = 20$ cm)

$\Rightarrow$ $AD^2 = 30^2 - 20^2 = 900 - 400 = 500$

$\Rightarrow$ $AD = \sqrt{500} = 10\sqrt{5}$ cm

$\therefore$ Area of an isosceles triangle $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 40 \times 10\sqrt{5} = 200\sqrt{5}$ cm^2

7. The sides of triangle are 8 cm, 15 cm and 17 cm. Find the area. **[CBSE 2016]**

Sol. Given $a = 8$ cm, $b = 15$ cm and $c = 17$ cm

The semi-perimeter of triangle, $s = \frac{a+b+c}{2}$

$\Rightarrow$ $s = \frac{8+15+17}{2} = \frac{40}{2} = 20$ cm

Using Heron's formula,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{20(20-8)(20-15)(20-17)}$

$= \sqrt{20 \times 12 \times 5 \times 3} = \sqrt{5 \times 4 \times 4 \times 3 \times 5 \times 3} = \sqrt{\underline{5 \times 5} \times \underline{4 \times 4} \times \underline{3 \times 3}} = 5 \times 4 \times 3 = 60$ cm^2

8. Find the perimeter of an isosceles right-angled triangle having an area of 5000 m^2. (Use $\sqrt{2} = 1.41$) **[CBSE 2015]**

Sol. Let ΔABC be an isosceles right-angled triangle in which AB = BC = x m.

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height

$\Rightarrow$ $5000 = \frac{1}{2} \times x \times x$ (Given area = 5000 m^2)

$\Rightarrow$ $x^2 = 5000 \times 2 = 10000$

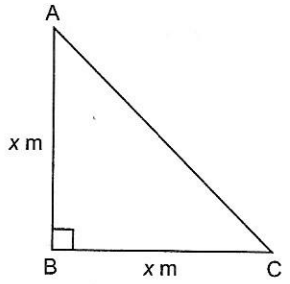
$\Rightarrow$ $x = \sqrt{10000} = 100$ m

Using Pythagoras Theorem in an isosceles right-angled ΔABC, we have

$AC^2 = AB^2 + BC^2 = x^2 + x^2 = 2x^2$

$\therefore$ $AC = x\sqrt{2} = 100\sqrt{2}$ m

$= 100 \times 1.41 = 141$ m



$\therefore$ Perimeter of an isosceles right-angled ΔABC = sum of all sides

$= AB + BC + AC$

$= 100 + 100 + 141 = 341$ m

9. The sides of triangle are 100 m, 120 m and 140 m. Find its area. (Use $\sqrt{6} = 2.45$) [CBSE 2016]

Sol. Given $a = 100$ m, $b = 120$ m, $c = 140$ m

$\therefore$ Perimeter of triangle, $2s = a + b + c$

$\Rightarrow$ $2s = 100 + 120 + 140 = 360$ m

$\Rightarrow$ Semi-perimeter, $s = \frac{360}{2} = 180$ m

Using Heron's formula,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

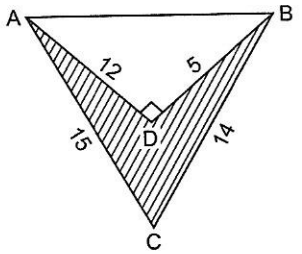
$= \sqrt{180(180-100)(180-120)(180-140)} = \sqrt{180 \times 80 \times 60 \times 40}$

$= \sqrt{60 \times 3 \times 40 \times 2 \times 60 \times 40} = \sqrt{60 \times 60 \times 40 \times 40 \times 6}$

$= 60 \times 40 \times \sqrt{6} = 2400 \times 2.45 = 5880$ m^2

Short Answer Type Questions II [3 Marks]

10. Find the area of shaded region in the given figure. (All measurements are in cm) [CBSE 2014]



Sol. Area of right-angled $\Delta ADB = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times BD \times AD$ (Base = BD, height = AD)

$= \frac{1}{2} \times 5 \times 12 = 30$ cm^2

Using Pythagoras theorem in right-angled ΔADB, we have

$AB^2 = AD^2 + BD^2 = 12^2 + 5^2 = 144 + 25 = 169$

$\therefore$ $AB = \sqrt{169} = 13$ cm

Now, in ΔABC, $AB = 13$ cm, $AC = 15$ cm and $BC = 14$ cm

$\therefore$ Perimeter of triangle, $2s = AB + BC + AC = 13 + 14 + 15 = 42$ cm

$\therefore$ Semi-perimeter, $s = \frac{42}{2} = 21$ cm

Using Heron's formula,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21(21-13)(21-15)(21-14)}$

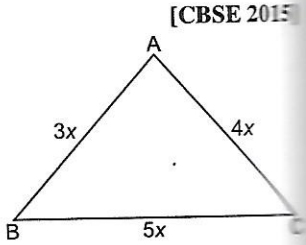
$= \sqrt{21 \times 8 \times 6 \times 7} = \sqrt{7 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7}$

$= \sqrt{7 \times 7 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2} = 7 \times 3 \times 2 \times 2 = 84$ cm^2

$\therefore$ Area of shaded portion $=$ ar(ΔABC) $-$ ar(ΔADB) $= 84 - 30 = 54$ cm^2.

11. The perimeter of a triangular garden is 900 cm and its sides are in the ratio 3 : 5 : 4. Using Heron's formula, find the area of triangular garden. [CBSE 2015]

Sol. Suppose that the sides of a triangular garden (in cm) are $3x$, $5x$ and $4x$.



Perimeter of triangular garden $= 900$ cm

$\Rightarrow$ $900 = 3x + 5x + 4x$

$\Rightarrow$ $900 = 12x$

$\Rightarrow$ $x = \frac{900}{12} = 75$ cm

So, the sides of triangular garden are 3×75 cm, 5×75 cm and 4×75 cm, i.e. 225 cm, 375 cm and 300 cm.

Now, we have semi-perimeter, $s = \frac{225+375+300}{2}\text{ cm} = \frac{900}{2} = 450\text{ cm}$

Using Heron's formula,

$$\text{Area of triangular garden} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{450(450-225)(450-375)(450-300)}$$
$$= \sqrt{450\times225\times75\times150} = \sqrt{225\times2\times225\times75\times75\times2}$$
$$= \sqrt{225\times225\times75\times75\times2\times2}$$
$$= 225\times75\times2 = 33{,}750\text{ cm}^2 = 3.375\text{ m}^2$$

12. Find the area of a triangle whose perimeter is 180 cm and its two sides are 80 cm and 18 cm. Calculate the altitude of triangle corresponding to its shortest side. **[CBSE 2015]**

Sol. Given $a = 80$ cm and $b = 18$ cm

$$\text{Perimeter of triangle} = a + b + c$$
$$\Rightarrow \quad 180 = 80 + 18 + c$$
$$\therefore \quad c = 180 - 98 = 82\text{ cm}$$

and semi-perimeter, $s = \frac{\text{Perimeter}}{2} = \frac{180}{2} = 90\text{ cm}$

Using Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{90(90-80)(90-18)(90-82)}$$
$$= \sqrt{90\times10\times72\times8} = \sqrt{10\times9\times10\times9\times8\times8}$$
$$= \sqrt{10\times10\times9\times9\times8\times8} = 10\times9\times8 = 720\text{ cm}^2$$

The shortest side of triangle = 18 cm

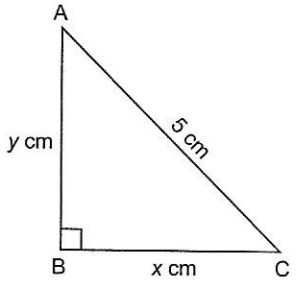
$$\therefore \quad \text{Area of triangle} = \frac{1}{2}\times\text{base}\times\text{altitude}$$
$$720 = \frac{1}{2}\times18\times h$$
$$\therefore \quad h = \frac{720}{9} = 80\text{ cm}$$

$\therefore$ Altitude of triangle corresponding to its shortest side (18 cm) is 80 cm.

Long Answer Type Questions [4 Marks]

13. The perimeter of a right-angled triangle is 12 cm and its hypotenuse is of length 5 cm. Find the other two sides and calculate its area. Verify the result using Heron's formula. **[HOTS]**

Sol. Let ΔABC be the right-angled triangle.



Suppose $AB = y$ cm and $BC = x$ cm

Now, $x + y + 5 = 12$ [Given perimeter of triangle is 12 cm]

$\Rightarrow \quad x + y = 7 \quad ...(i)$

Using Pythagoras theorem in right-angled ΔABC, we get

$$x^2 + y^2 = 25 \quad ...(ii)$$

Squaring (i) both sides, we get $(x+y)^2 = 7^2$

$$\Rightarrow \quad x^2 + y^2 + 2xy = 49$$
$$\Rightarrow \quad 25 + 2xy = 49$$
$$\Rightarrow \quad xy = 12 \quad ...(iii)$$

$$\therefore \quad \text{Area of right-angled } \Delta ABC = \frac{1}{2}\times\text{base}\times\text{height} = \frac{1}{2}\times x\times y = \frac{1}{2}\times12 = 6\text{ cm}^2$$

Now, consider $(x-y)^2 = (x+y)^2 - 4xy = 49 - 4\times12 = 1$

$\Rightarrow \quad x - y = \pm 1$

If $x + y = 7$ and $x - y = 1$, we get $x = 4, y = 3$

If $x + y = 7$ and $x - y = -1$, we get $x = 3, y = 4$

Therefore, length of the other two sides of triangle are 3 cm and 4 cm.

Verification of Area of ΔABC by Heron's formula:

Semi-perimeter, $s = \frac{3+4+5}{2} = 6$ cm

Using Heron's formula,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{6(6-3)(6-4)(6-5)} = \sqrt{6\times3\times2\times1} = \sqrt{6\times6} = 6\text{ cm}^2$

which is same as obtained earlier.

Hence, result is verified.

14. How much area of triangle will increase in percentage, if each side of the triangle is doubled? [HOTS]

Sol. Let a, b, c be the sides of the triangle.

Let its semi-perimeter be s_1

$\therefore \quad s_1 = \frac{a+b+c}{2}$

Using Heron's formula,

$$\text{Area of triangle, } A_1 = \sqrt{s(s-a)(s-b)(s-c)} \Rightarrow A_1 = \sqrt{s_1(s_1-a)(s_1-b)(s_1-c)}$$

When the sides of triangle are doubled, i.e. $2a, 2b, 2c$, then its

$$\text{semi-perimeter, } s_2 = \frac{2a+2b+2c}{2} = 2\left(\frac{a+b+c}{2}\right) = 2s_1$$

Using Heron's Formula,

$$\text{Area of triangle, } A_2 = \sqrt{s_2(s_2-2a)(s_2-2b)(s_2-2c)}$$

$\Rightarrow \quad A_2 = \sqrt{2s_1(2s_1-2a)(2s_1-2b)(2s_1-2c)} \quad (\because s_2 = 2s_1)$

$$= 4\sqrt{s_1(s_1-a)(s_1-b)(s_1-c)}$$

$\Rightarrow \quad A_2 = 4A_1 = 4 \times \text{Area of original triangle}$

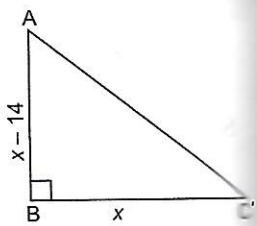
Hence, percentage increase in area $= \frac{\text{Increase in area}}{\text{Original area}} \times 100 = \frac{A_2 - A_1}{A_1} \times 100 = \left(\frac{A_2}{A_1} - 1\right) \times 100$

$$= (4-1) \times 100 = 300\%$$

15. The difference between the two adjoining sides containing right angle of a right-angled triangle is 14 cm. The area of triangle is 120 cm^2. Verify this area by using Heron's formula. [HOTS]

Sol. Let ΔABC be the right-angled triangle with $\angle B = 90°$,

Let $BC = x$ cm and $AB = (x-14)$cm



$\therefore \quad$ Area of right-angled $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BC \times AB$

$\Rightarrow \quad 120 = \frac{1}{2} \times x \times (x-14) \Rightarrow 240 = x^2 - 14x$

$\Rightarrow \quad x^2 - 14x - 240 = 0 \Rightarrow x^2 - 24x + 10x - 240 = 0$

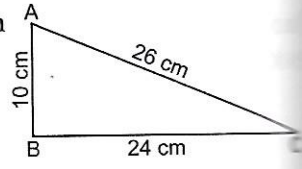
$\Rightarrow \quad x(x-24) + 10(x-24) = 0 \Rightarrow (x-24)(x+10) = 0$

$\Rightarrow$ either $x - 24 = 0$ or $x + 10 = 0 \Rightarrow$ either $x = 24$ or $x = -10$

Since length cannot be negative, so by ignoring $x = -10$, we get

$BC = x = 24$ cm and $AB = x - 14 = 24 - 14 = 10$ cm

and hypotenuse, $AC = \sqrt{AB^2 + BC^2} = \sqrt{10^2 + 24^2} = \sqrt{100 + 576} = \sqrt{676} = 26$ cm



Verification of Area of ΔABC by Heron's formula:

Let $a = 24$ cm, $b = 10$ cm, $c = 26$ cm

$\therefore \quad$ Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{24+10+26}{2} = \frac{60}{2} = 30$ cm

Using Heron's formula,

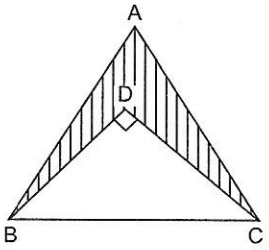
Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{30(30-24)(30-10)(30-26)}$

$= \sqrt{30\times6\times20\times4} = \sqrt{6\times5\times6\times5\times4\times4} = \sqrt{6\times6\times5\times5\times4\times4}$

$= 6 \times 5 \times 4 = 120\text{ cm}^2$ Hence verified

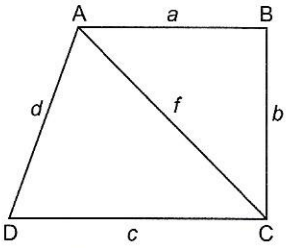
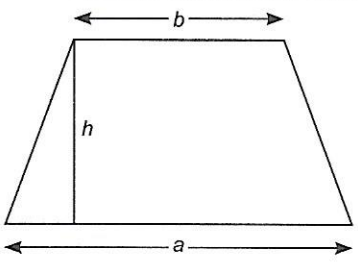
PRACTICE QUESTIONS BASED ON EXERCISE 12.1

1. State two situations in day-to-day life when we have to find the area.
2. Is it possible to find the area of triangle whose height cannot be determined easily? If yes, how?
3. Area of an isosceles triangle of sides a cm, b cm and b cm is $\frac{1}{4}b\sqrt{4a^2 - b^2}$ cm^2. State true or false. Justify your answer.
4. The perimeter of an equilateral triangle is 60 cm. Find its area. (Use $\sqrt{3} = 1.73$)
5. Using Heron's formula, find the area of an equilateral triangle of side x cm.
6. Find the area of triangle whose sides are 18 cm, 24 cm and 30 cm.
7. Using Heron's formula, find the area of an equilateral triangle whose perimeter is 162 cm. (Use $\sqrt{3} = 1.73$)
8. The base of an isosceles triangle is 10 cm and one of its equal sides is 13 cm. Find its area using Heron's formula.
9. Find the area of triangle with sides 35 cm, 54 cm and 61 cm. (Use $\sqrt{5} = 2.236$) **[CBSE 2013]**
10. In the given figure, ΔABC is an equilateral triangle with side 10 cm and ΔDBC is right angled triangle with $\angle D = 90°$. If BD = 6 cm, then find the area of the shaded portion. (Use $\sqrt{3} = 1.732$) **[CBSE 2013; HOTS]**



11. The sides of a triangle are in the ratio 13 : 14 : 15 and its perimeter is 84 cm. Find the area of the triangle.
12. The perimeter of an isosceles triangle is 32 cm. The ratio of equal side to the base is 3 : 2 Using Heron's formula, find the area of triangle. **[HOTS]**
13. The sides of triangle are 8 cm, 15 cm and 17 cm. Find the area of the triangle. Also, find the length of the altitude drawn on the side with length 17 cm.
14. The sides of triangular field are 41 m, 40 m and 900 cm. Each rose bed requires 180 cm^2 of space. How many rose beds can be prepared in the field?
15. A gardener has to put double fence all around a triangular field with sides 120 m, 80 m and 60 m. In the middle of each of the sides, there is a gate of width 10 m.
    (*i*) Find the length of wire needed for fencing.
    (*ii*) Find the cost of fencing at the rate of ₹ 6 per metre.
    (*iii*) Find the area of triangular field.

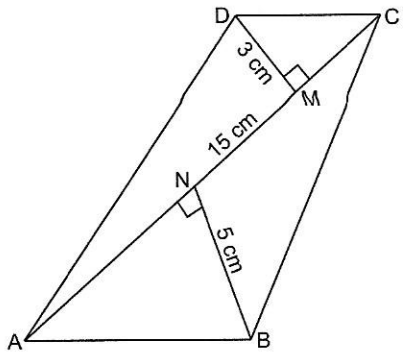
Application of Heron's Formula in Finding Areas of Quadrilaterals

• A quadrilateral whose sides are a, b, c, d and one diagonal 'f' are given.		∴ Area of quadrilateral ABCD = ar(ΔABC) + ar(ΔADC) **Note:** Apply Heron's formula for finding the areas of both triangles.
• A trapezium with sides a and b and the distance between its two parallel sides is h.		∴ Area of trapezium $= \frac{1}{2} \times (a + b) \times h$

SOLVED QUESTIONS BASED ON EXERCISE 12.2

Short Answer Type Questions I [2 Marks]

1. Find the area of quadrilateral ABCD as shown in the figure:



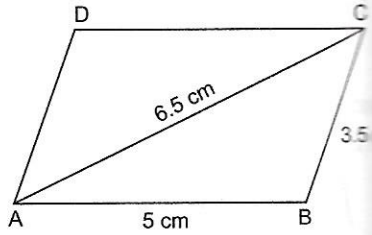
Sol. Area of quadrilateral ABCD = ar(ΔABC) + ar(ΔADC) = $\frac{1}{2}$ ×AC × BN + $\frac{1}{2}$ × AC × DM

$$= \frac{1}{2} \times 15 \times 5 + \frac{1}{2} \times 15 \times 3$$

$$= \frac{15}{2} \times (5 + 3) = \frac{15}{2} \times 8 = 60 \text{ cm}^2$$

2. Two adjacent sides of a parallelogram measures 5 cm and 3.5 cm. One of its diagonal measures 6.5 cm. Find the area of the parallelogram.

Sol. Let ABCD be the parallelogram with AB = 5 cm, BC = 3.5 cm and AC = 6.5 cm as shown in figure.



$\therefore$ Semi-perimeter of ΔABC, $s = \frac{a+b+c}{2}$

$$= \frac{5+3.5+6.5}{2} = \frac{15}{2} = 7.5 \text{ cm}$$

Using Heron's formula,

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{7.5(7.5-5)(7.5-3.5)(7.5-6.5)}$$

$$= \sqrt{7.5 \times 2.5 \times 4 \times 1}$$

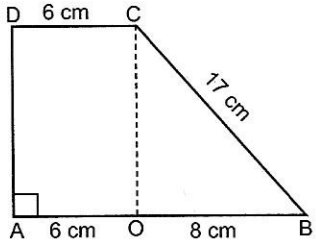
$$= \sqrt{7.5 \times 10} = \sqrt{75} = 5\sqrt{3} \text{ cm}^2$$

We know that the diagonal of a parallelogram divides it into two congruent triangles of equal area.

$\therefore$ Area of parallelogram ABCD = 2 × ar(ΔABC)

$$= 2 \times 5\sqrt{3} = 10\sqrt{3} \text{ cm}^2$$

3. Calculate the area of trepezium as shown in the figure: [CBSE 2



Sol. In ΔBOC, $\angle O = 90°$ as $CO \parallel AD$.

Using Pythagoras theorem in right-angled ΔBOC, we have

$$BC^2 = OC^2 + OB^2$$

$$\Rightarrow \quad 17^2 = OC^2 + 8^2$$

$$\Rightarrow \quad OC = \sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225}$$

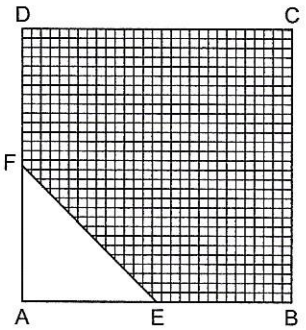
$$\therefore \quad OC = 15 \text{ cm}$$

$$\therefore \quad \text{Area of trapezium } ABCD = \frac{1}{2} \times (AB + CD) \times OC$$

$$= \frac{1}{2} \times (14 + 6) \times 15 \qquad (\because AB = AO + OB = 6 + 8 = 14 \text{ cm})$$

$$= \frac{1}{2} \times 20 \times 15 = 150 \text{ cm}^2$$

4. In the given figure, ABCD is a square of side 4 cm. E and F are mid-points of AB and AD respectively. Find the area of the shaded region. **[CBSE 2016]**



Sol. Area of square $ABCD = (\text{side})^2$ $(\because$ Side of square $= 4$ cm$)$

$$= 4^2 = 16 \text{ cm}^2$$

Area of right-angled $\Delta EAF = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AE \times AF$

$$= \frac{1}{2} \times 2 \times 2 \qquad (\because AE = \frac{1}{2}AB \text{ and } AF = \frac{1}{2}AD)$$

$$= 2 \text{ cm}^2$$

$\therefore$ Area of shaded region = Area of square $ABCD$ – Area of $\Delta EAF = 16 - 2 = 14 \text{ cm}^2$

5. Find the area of the parallelogram, whose one diagonal is 6.8 cm and the perpendicular distance from the opposite vertex is 7.5 cm. **[HOTS]**

Sol. We know that the diagonal of parallelogram divides it into two congruent triangles of equal area.

$$\therefore \quad \text{Area of one triangle} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 6.8 \times 7.5 = 25.5 \text{ cm}^2$$

$$\therefore \quad \text{Area of parallelogram} = 2 \times \text{Area of one congruent triangle}$$

$$= 2 \times 25.5 = 51 \text{ cm}^2$$

Short Answer Type Questions II [3 Marks]

6. A floor design is made on a floor of a room by joining four triangular tiles of dimensions 12 cm, 20 cm and 24 cm each. Find the cost of the tiles at the rate of ₹ $\sqrt{14}$ per cm^2. **[HOTS]**

Sol. Given $a = 12$ cm, $b = 20$ cm and $c = 24$ cm

$\therefore$ Semi-perimeter, $s = \frac{a+b+c}{2}$

$$\Rightarrow \quad s = \frac{12 + 20 + 24}{2} = \frac{56}{2} = 28 \text{ cm}$$

Using Heron's formula,

Area of one triangular tile $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{28(28-12)(28-20)(28-24)}$

$= \sqrt{28\times16\times8\times4} = \sqrt{14\times2\times4\times4\times4\times2\times4} = \sqrt{4\times4\times4\times4\times2\times2\times14}$

$= 4\times4\times2\times\sqrt{14} = 32\sqrt{14}$ cm^2

$\therefore$ Area of 4 such tiles $= 4 \times 32\sqrt{14} = 128\sqrt{14}$ cm^2

Cost of one tile $= ₹\ \sqrt{14}$ per cm^2

$\therefore$ Total cost of 4 tiles $= ₹\ \sqrt{14} \times 128\sqrt{14}$

$= ₹\ 128 \times 14 = ₹\ 1792$

Hence, cost of designing the floor $= ₹\ 1792$.

7. A forest reservoir is in the shape of quadrilateral whose sides taken in order are 9 m, 40 m, 15 m and 28 m. If the angle between first two sides is a right angle, find the area of a forest reservoir. [HOTS]

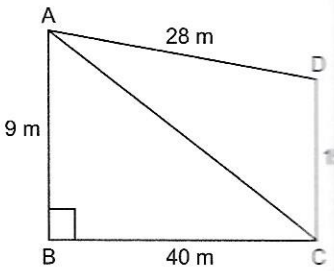
Sol. ΔABC is a right-angled triangle. So, by using Pythagoras theorem in right-angled ΔABC, we have

$AC^2 = AB^2 + BC^2$

$= 9^2 + 40^2$

$= 81 + 1600 = 1681$

$\therefore$ $AC = \sqrt{1681} = 41$ m



$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 40 \times 9 = 180$ m^2

Now, in ΔACD,

let $a = AC = 41$ m, $b = CD = 15$ m and $c = AD = 28$ m

Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{41+15+28}{2} = \frac{84}{2} = 42$ m

Using Heron's formula,

Area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-41)(42-15)(42-28)}$

$= \sqrt{42\times1\times27\times14} = \sqrt{14\times3\times3\times3\times3\times14} = \sqrt{14\times14\times3\times3\times3\times3} = 14\times3\times3 = 126$ m^2

$\therefore$ Area of quadrilateral ABCD $=$ ar(ΔABC) + ar(ΔACD)

$= 180 + 126 = 306$ m^2

8. Sanya has a piece of land which is in the shape of a rhombus. She wants her one daughter and one son to work on the land and produce different crops to suffice the needs of their family. She divided the land in two equal parts. If the perimeter of the land is 400 m and one of the diagonal is 160 m, how much area each of them will get?

Sol. Let ABCD be the field.

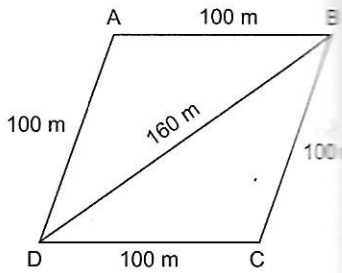
Perimeter of field $= 400$ m

So, each side $= 400$ m $\div 4 = 100$ m

i.e. $AB = AD = 100$ m

Let diagonal $BD = 160$ m

Then semi-perimeter of ΔABD is given by $s = \frac{100+100+160}{2} = 180$ m



Using Heron's formula,

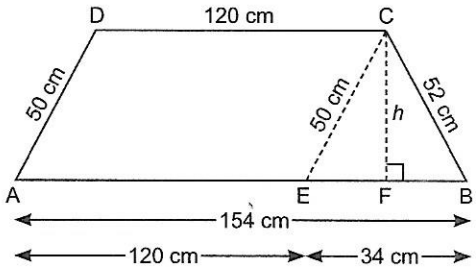
$$\text{Area of } \Delta ABD = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{180(180-100)(180-100)(180-160)}$$
$$= \sqrt{180 \times 80 \times 80 \times 20}\ \text{m}^2 = 4800\ \text{m}^2$$

Therefore, each of them will get an area of 4800 m^2 since ar(ΔABD) = ar(ΔBCD).

9. Two parallel sides of a trapezium are 120 cm and 154 cm and other sides are 50 cm and 52 cm. Find the area of trapezium. **[CBSE 2011]**

Sol. Let ABCD be a trapezium in which AB = 154 cm, CD = 120 cm, AD = 50 cm, BC = 52 cm.

Construction: Draw CE || AD and CF ⊥ AB.



Now, CD || AB and CE || DA

$\therefore$ AECD is a parallelogram.

$\Rightarrow$ CE = AD = 50 cm

and CD = AE = 120 cm

$\therefore$ BE = AB – AE = 154 – 120 = 34 cm

In ΔBEC, semi-perimeter, $s = \dfrac{a+b+c}{2}$

$\Rightarrow$ $s = \dfrac{50+52+34}{2} = \dfrac{136}{2} = 68$ cm

Using Heron's formula,

$$\text{Area of } \Delta CEB = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{68(68-50)(68-52)(68-34)} = \sqrt{68 \times 18 \times 16 \times 34}$$
$$= \sqrt{34 \times 2 \times 2 \times 9 \times 4 \times 4 \times 34} = \sqrt{34 \times 34 \times 2 \times 2 \times 3 \times 3 \times 4 \times 4}$$
$$= 34 \times 2 \times 3 \times 4$$
$$= 34 \times 24 = 816\ \text{cm}^2$$

But $\text{Area of } \Delta CEB = \dfrac{1}{2} \times \text{base} \times \text{height}$

$\Rightarrow$ $816 = \dfrac{1}{2} \times BE \times CF$

$\therefore$ $816 = \dfrac{1}{2} \times 34 \times CF$

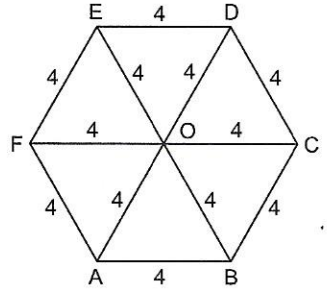
$\Rightarrow$ $CF = \dfrac{816}{17} = 48$ cm

$\therefore$ $\text{Area of trapezium ABCD} = \dfrac{1}{2} \times (\text{Sum of parallel sides}) \times \text{height} = \dfrac{1}{2} \times (AB + CD) \times CF$

$$= \frac{1}{2} \times (154 + 120) \times 48 = \frac{1}{2} \times 274 \times 48 = 6576\ \text{cm}^2$$

10. Find the area of a regular hexagon whose one side is 4 units.

Sol. The six sides of regular hexagon are of equal length. The point of intersection of its diagonal divides it into six equilateral triangles each of side 4 units as shown in figure.



$\therefore$ Area of an equilateral triangle each side of 'a' units $= \dfrac{\sqrt{3}}{4} \times a^2$

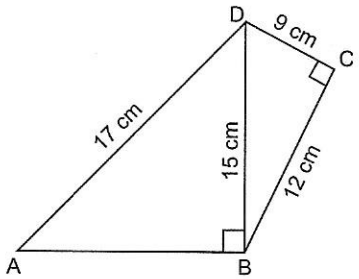
$\therefore$ $\text{Area of } \Delta OAB = \dfrac{\sqrt{3}}{4} \times 4^2 = 4\sqrt{3}$ sq. units

$\therefore$ $\text{Area of hexagon ABCDEF} = 6 \times \text{ar}(\Delta OAB)$

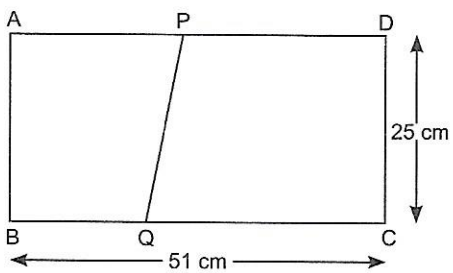
$$= 6 \times 4\sqrt{3} = 24\sqrt{3}\ \text{sq. units}$$

PRACTICE QUESTIONS BASED ON EXERCISE 12.2

1. Find the area of quadrilateral ABCD as shown in figure given below:



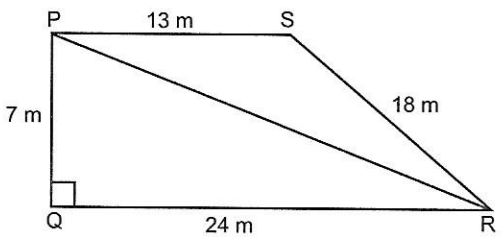
2. The length of two adjacent sides of a parallelogram are respectively 51 cm and 37 cm. One of its diagonal is 20 cm. Find the area of the parallelogram.
3. Calculate the area of quadrilateral PQRS when length of diagonal PR = 10 cm and length of perpendiculars from Q and S on PR be 5 cm and 6 cm respectively.
4. The dimensions of a rectangle ABCD are 51 cm × 25 cm. A trapezium PQCD with its parallel sides QC and PD in the ratio 9 : 8, is cut off from the rectangle as shown in the figure. If the area of the trapezium PQCD is $\frac{5}{6}$th part of the area of the rectangle, find the lengths QC and PD. **[HOTS]**



5. A rhombus shaped sheet with perimeter 40 cm and one diagonal 12 cm is painted on both sides of the rate of ₹ 6 per cm^2. Find the cost of painting.
6. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 126 cm, 120 cm and 130 cm and the parallelogram stands on the base 120 cm, find the corresponding height of the parallelogram. **[CBSE 2014]**
7. Find the area of quadrilateral ABCD in which AB = 50 m, BC = 60 m, CD = 30 m, DA = 90 m and BD = 70 m. (Use $\sqrt{5}$ = 2.24 and $\sqrt{11}$ = 3.32) **[CBSE 2014]**
8. Anurag makes a kite using red and yellow piece of paper. Red piece of paper is cut in the shape of square with diagonal 30 cm. At one of the vertex of this square, a yellow paper with the shape of an equilateral triangle of side such that $a^2 = 32\sqrt{3}$ is attached to give the shape of a kite. Find the total area of paper required to make the kite. **[HOTS]**
9. The shape of cross-section of a canal is a trapezium. The canal is 10 m wide at the top and 6 m wide at the bottom and the area of cross section is 72 m^2. Find its depth.
10. The area of trapezium is 475 cm^2 and its height is 19 cm. Find the length of its two parallel sides, if one side is 40 cm greater than the other. **[HOTS]**

Value Based Questions

1. The students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes PQ, QR and RP; while the other group walked through PR, RS and SP as shown in figure:



These two groups cleaned the area enclosed within their lanes. If PQ = 7 m, QR = 24 m, RS = 18 m, SP = 13 m and ∠Q = 90°;

(*i*) Which group cleaned more area and by how much?

(*ii*) Find the total area cleaned by the students, (neglecting the width of the lane).

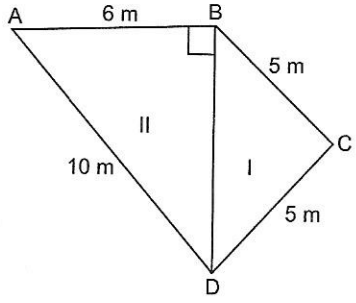
(*iii*) What values are depicted here by these students? (Use $\sqrt{14}$ = 3.74)

2. Rangaswami has a vacant triangular land near his house with the sides 13m, 14m and 15m. He wants to help the poor people by growing some vegetables in that vacant triangular field.

(*i*) If the quantity of vegetables which grows per m^2 is enough for 6 persons for one day, then find the number of persons who can take benefit out of it per day.

(*ii*) What values are depicted by Rangaswami?

3. A villager has a field in the form of a quadrilateral with sides 6 m, 10 m, 5 m and 5 m. He divided the land into two parts along one of its diagonal as shown in figure. He gives the part I to his son, while the part II is equally divided amongst his two daughters. He wants his children to work together on the land and produces different crops.



(*i*) Is this distribution fair? Justify it.

(*ii*) What values are depicted here?

4. The Sarpanch of a village Padampur requested one of his villagers to donate a 6 m wide land adjusted to his 132.8 m long side of his right triangular plot outside the village. The other sides of the plot are 123 m and 50 m. On his donated land, the Sarpanch wants to construct a link road which provides the connectivity with the other villages and towns. The villager agreed at once.

(*i*) Find the area of triangular plot remaining with the villager.

(*ii*) What are the values involved here?

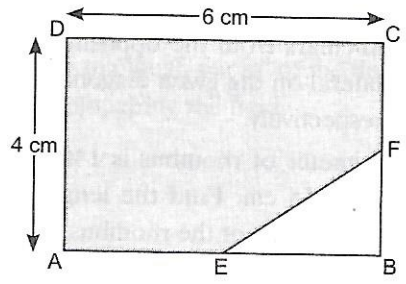
INTEGRATED EXERCISE

Very Short Answer Type Questions [1 Mark]

1. An isosceles right-angled triangle has an area 32 cm^2. Find the length of its hypotenuse.
2. Find the perimeter of an equilateral triangle, if its area is $64\sqrt{3}$ cm^2.
3. Calculate the area of an isosceles triangle with two equal sides as 5 cm and unequal side as 8 cm.
4. Find the area of an equilateral triangle with side 6 cm.
5. The perimeter of an isosceles right-angled triangle is 41 cm. If its hypotenuse is 17 cm, find its area.
6. In a triangle, the sides are given as 11 cm, 12 cm and 13 cm. The length of the altitude is 10.25 cm corresponding to the side of length 12 cm. Find its area. **[NCERT Exemplar]**

Short Answer Type Questions I [2 Marks]

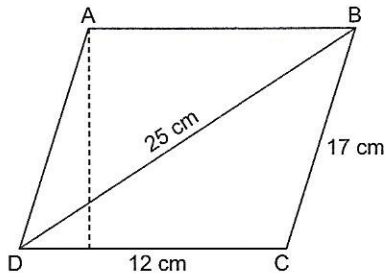
7. Find the area of triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm.
8. The semi-perimeter of triangle is 96 cm and its sides are in the ratio 3 : 4 : 5. Find the area of the triangle.
9. The area of an isosceles triangle is $8\sqrt{15}$ cm^2. If the base is 8 cm, find the length of each of its equal sides.
10. If the side of rhombus is 10 cm and one of its diagonal is 16 cm, then find its area.
11. Write whether the given statement is true or false. "The base and corresponding altitude of a parallelogram are 10 cm and 3.5 cm respectively. The area of parallelogram is 30 cm^2. Justify your answer. **[NCERT Exemplar]**
12. Find the cost of levelling the ground in the form of triangle having the sides 26 m, 28 m and 30 m at the rate of ₹ 3 per m^2.
13. In the given figure, ABCD is a rectangle of dimensions 4 cm and 6 cm. E and F are mid-points of AB and BC respectively. Find the area of the shaded portion. **[CBSE 2014]**



Short Answer Type Questions II [3 Marks]

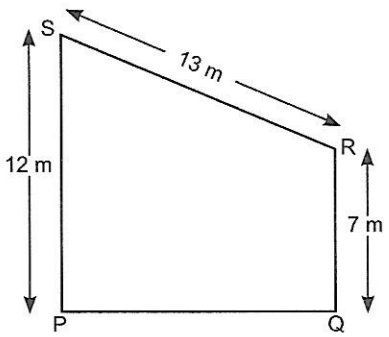
14. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisement yields an earning of ₹ 2000 per m^2 per year. A company hired one of its walls for 6 months. How much rent did it pay? **[NCERT Exemplar]**

15. Find the area of parallelogram given in the figure. Also, find the length of the altitude from vertex A on the side DC. **[NCERT Exemplar]**



16. The sides of quadrilateral ABCD are 6 cm, 8 cm, 12 cm and 14 cm (taken in order) respectively and the angle between first two sides is a right angle. Find its area. **[NCERT Exemplar]**

17. Find the area of trapezium PQRS with height PQ in the figure given below. **[NCERT Exemplar]**



18. The sides of triangle are in the ratio 25 : 17 : 12 and its perimeter is 540 m. Find the area of the triangle.

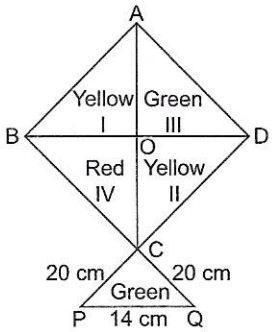
19. Find the area of quadrilateral when one of the diagonal measures 50 cm and length of the perpendiculars from the opposite vertices of the quadrilateral on the given diagonal are 10 cm and 20 cm respectively.

20. The perimeter of rhombus is 146 cm. One of its diagonal is 55 cm. Find the length of the other diagonal and area of the rhombus.

21. There is a quadrilateral field ABCD in which AB = 3 m, BC = 4 m, CD = 6 m, DA = 5 m and diagonal AC = 5 m. The green grass of this field is available for grazing. Find out how many cows can graze the whole field in a day, if one cow can graze one m^2 per day.

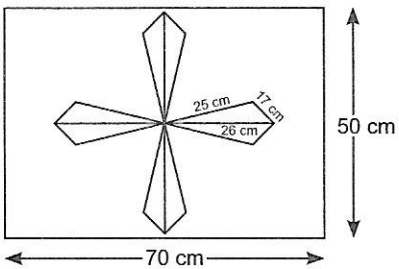
Long Answer Type Questions [4 Marks]

22. How much paper of each shade is needed to make a kite given in figure, in which ABCD is a square with diagonal 44 cm. (Use $\sqrt{39} = 6.24$) **[NCERT Exemplar]**



23. The perimeter of triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle. **[NCERT Exemplar]**

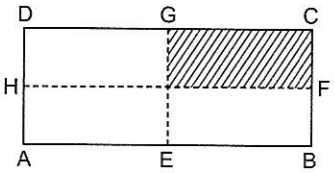
24. A design is made on a rectangular tile of dimensions 50 cm × 70 cm as shown in figure given below. The design shows 8 triangles, each of sides 26 cm, 17 cm and 25 cm. Find the total area of the design and the remaining area of the tile. **[NCERT Exemplar]**



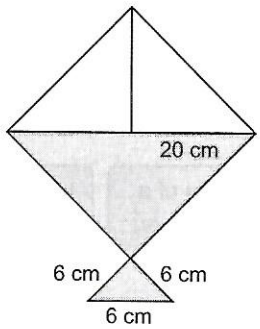
25. Kamla has a triangular field with sides 240 m, 200 m, 360 m, where she grew wheat. In another triangular field with sides 240 m, 320 m, 400 m adjacent to the previous field, she wanted to grow potatoes and onions. She divided the field in two parts by joining the mid-point of the longest side to the opposite vertex and grew potatoes in one part and onions in the other part. How much area (in hectares) has been used for wheat, potatoes and onions? (1 hectare = 10000 m^2)

ASSESS YOURSELF

1. The base of a right-angled triangle is 16 cm and hypotenuse is 34 cm. Find the area of triangle.
2. Find the half of the perimeter of an equilateral triangle whose area is $4\sqrt{3}$ cm^2.
3. An advertisement board is in the form of an equilateral triangle of perimeter 240 cm. Find the area of the board using Heron formula. (Use $\sqrt{3} = 1.73$)
4. Find the area of triangle whose sides are 5 cm, 7 cm and 9 cm.
5. A student is given three sticks of length 15 cm, 8 cm, 5 cm respectively. His friend asked him to make a triangle with the help of these sticks and then find its area.
6. The perimeter of a triangular field is 240 cm. If two of its sides are 78 cm and 50 cm, find the length of perpendicular on the sides of length 50 cm from the opposite vertex.
7. In the given figure, ABCD is a rectangle of dimensions 6 cm and 10 cm. E, F, G and H are the mid-points of the sides taken in order. Find the area of the shaded portion.



8. Find the area of an isosceles triangle where perimeter is 11 cm and base is 5 cm.
9. The edges of a triangular board are 6 cm, 8 cm and 10 cm. Find the cost of painting it at ₹ 0.09 per cm^2.
10. From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 m. Find the area of the triangle.
11. A field in the form of a parallelogram has sides 60 m and 40 m and one of its diagonal is 80 m long. Find the area of parallelogram.
12. The area of trapezium is 475 cm^2 and height is 19 cm. If one side of the trapezium is 4 cm greater than the other parallel side, then find their length.
13. If each side of a triangle is doubled, then find the ratio of area of new triangle thus formed and the original triangle.
14. A kite in the shape of a square of diagonal 20 cm and an equilateral triangle of base 6 cm is made of three different shades, as shown. How much paper for the shaded portion has been used?



15. The parallel sides of the trapezium field are 80 m and 20 m. These sides meet the third side at right angles. The length of the fourth side is 90 m. If its costs ₹ 5 to plough per m^2 of the field, find the total cost of ploughing the field.